CLASS - X
MATHEMATICS

Trigonometric Inverse Circular Function

1. Solve : $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31},(x>0)$.

Ans. Solution: $tan^{-1}(x+1)+tan^{-1}(x-1)=tan^{-1}\frac{8}{31}$

or, $tan^{-1}\frac{(x+1)+(x-1)}{1-(x+1)(x-1)}=tan^{-1}\frac{8}{31}$

or, $tan^{-1}\frac{2x}{1-(x^2-1)}=tan^{-1}\frac{8}{31}$

$\therefore\ \frac{2x}{2-x^2}=\frac{8}{31}$

or, $\frac{x}{2-x^2}=\frac{4}{31}$

or, $31x=8-4x^2$

or, $4x^2+31x-8=0$

or, $4x^2+32x-x-8=0$

or, $4x(x+8)-1(x+8)=0$

or, $(x+8)(4x-1)=0$

$\therefore\ x=-8,\frac{1}{4}$

$\because\ x>0$ $\therefore\ x\neq -8$ Therefore the required solution is $x=\frac{1}{4}$

2. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\Pi}{2}$ and $x+y+z=\sqrt{3}$ then prove that $x=y=z$.

Ans. Solution: $tan^{-1}x+tan^{-1}y+tan^{-1}z=\frac{\Pi}{2}$

or, $tan^{-1}x+tan^{-1}y=\frac{\Pi}{2}-tan^{-1}z$

or, $tan^{-1}\left(\frac{x+y}{1-xy}\right)=\frac{\Pi}{2}-tan^{-1}z$

or, $\frac{x+y}{1-xy}=tan\left(\frac{\Pi}{2}-tan^{-1}z\right)$

$$or; \quad \frac{x+y}{1-xy} = cot(tan^{-1}z)$$

$$or; \quad \frac{x+y}{1-xy} = cot(cot^{-1}\frac{1}{z})$$

$$or; \quad \frac{x+y}{1-xy} = \frac{1}{z}$$

$$or; \quad xy + yz + zx = 1$$

Again, $x + y + z = \sqrt{3}$

$$or; \quad x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 3$$

$$or; \quad x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 3(xy + yz + zx) \qquad \because\ xy + yz + zx = 1$$

$$or; \quad x^2 + y^2 + z^2 - xy - yz - zx = 0$$

$$or; \quad 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx = 0$$

$$or; \quad (x-y)^2 + (y-z)^2 + (z-x)^2 = 0$$

$\because$ Sum of the squares of three real quantities is zero

then each quantity will be zero separately

$\therefore$ $x - y = 0$, $y - z = 0$ and $z - x = 0$

$\therefore$ $x = y$, $y = z$ and $z = x$

$\therefore$ $x = y = z$ (Proved)

3. **Solution :** $\mathbf{tan - 1\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\Pi}{4}}$

Ans. Solution :

$$tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\Pi}{4}$$

$$or; \quad tan^{-1}\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2}.\frac{x+1}{x+2}} = \frac{\Pi}{4}$$

$$or; \quad \frac{x^2 + x - 2 + x^2 - x - 2}{x^2 - 4 - (x^2 - 1)} = \tan\frac{\Pi}{4} = 1$$

$$or; \quad 2x^2 - 4 = -3$$

$$or; \quad 2x^2 = 1$$

$$or; \quad x = \pm\frac{1}{\sqrt{2}}$$

Therefore the required solutions are $x = \pm\frac{1}{\sqrt{2}}$.

4. Solution : $\tan^{-1}2x+\tan^{-1}3x=\frac{\Pi}{4}$.

Ans. Solution :

$$tan^{-1}2x+tan^{-1}3x=\frac{\Pi}{4}$$

or, $$tan^{-1}\frac{2x+3x}{1-2x.3x}=\frac{\Pi}{4}$$

or, $$\frac{5x}{1-6x^2}=tan\frac{\Pi}{4}=1$$

or, $$5x=1-6x^2$$

or, $$6x^2+5x-1=0$$

or, $$(x+1)(6x-1)=0$$

$\therefore$ $x=-1,\frac{1}{6}$

$x=-1$ does not satisfy the given equation

$\therefore$ The required solution is $x=\frac{1}{6}$.